Finite-Size Bosonization of the Fractional Quantum Hall Edge: Optical Response of an Isolated Ring and DC Transport through an Antidot

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Fractional quantum Hall (FQH) edge states are a prototypical realization of chiral Luttinger liquids, and their low-energy dynamics are commonly described within bosonization theory [1-3]. While this framework has been highly successful for extended edges, many experimentally relevant systems involve mesoscopic geometries, such as ring-shaped edges and antidot structures, in which finite circumference, compactness, and zero modes become essential to observable quantities. In particular, isolated ring geometries provide a natural setting for studying the optical response of a finite FQH edge [4,5], while antidot devices offer a controlled platform in which charging effects, enclosed magnetic flux, discrete edge excitations, and anyon-related phase effects may influence measurable transport characteristics [6-9]. A theoretical framework that consistently incorporates such finite-size effects as well as finite-temperature effects is therefore important for connecting effective edge theory with measurable signatures in confined FQH systems.

In this work, we develop a finite-size bosonization framework for the fractional quantum Hall edge at finite temperatures. In contrast to the standard treatment for an infinitely extended edge, a finite geometry requires a careful treatment of compactification and zero modes, which play a central role in mesoscopic observables. By expressing finite-size and finite-temperature contributions exactly in terms of Jacobi theta functions, our formulation provides a unified low-energy description of edge excitations in closed geometries and allows both linear and nonlinear optical responses, as well as transport observables, to be evaluated in closed analytic form.

Using this approach, we study the optical response of an isolated ring, including both the linear electrical polarization susceptibility and the nonlinear optical response, and analyze how the discrete mode structure of the finite edge appears in these observables. For the isolated ring, we find that while the linear optical response exhibits peak shifts reflecting quasiparticle tunneling, the nonlinear optical response is nonzero only in the presence of tunneling, thereby isolating the tunneling-induced contribution. The combination of these two responses is expected to provide a route to extracting the anyonic statistics of fractional quantum Hall quasiparticles. For the antidot geometry, we demonstrate that the crossover from Coulomb-blockade-dominated sequential tunneling to coherent resonant tunneling and cotunneling can be systematically controlled by tuning the antidot size.

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