Revisiting the classical-quantum correspondence

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The most common quantum-classical correspondence follows from the Ehrenfest theorem, which relates the expectation values for the position and momentum to the expectation value of their corresponding Hamilton equation counterparts. In other words, the classical equations of motion hold after taking a quantum average of them. But, these equations can be misleading—for example, in a definite energy state of the harmonic oscillator, the averages are exactly zero, corresponding to the fact that these are stationary states. Instead, if we compare coherent states (a displaced ground state), their expectation value satisfies the same equation of motion as the classical system. The quantum system also has uncertainty, which the classical system does not. Key to this work is that the classical and quantum potentials are *identical.* But, then one finds the ground-state energy for the quantum system is different from the lowest energy state of the classical system.

We approach this differently [1.2]. If one takes the classical Hamilton equations of motion and decouples them by forming A=p+f(x), then the classical equation of motion can be solved exactly for all exactly sovlable quantum systems. This produces a new way to solve for Kepler orbits that is remarkably simple and apparfently unknown prior to this work. Furthermore, if we quantize position and momentum and form A^*A, we find it creates a Hamiltonian with an identical form of the equation of motion for the ladder operators. But, the quantum potential has a quantum correction relative to the classical potential. Now, however, the ground-state energy is identical to the lowest-energy state of the classical system. Furthermore, one can compare the orbits of position and momentum for the classical and quantum systems and in general there are now differences (for the simple harmonic oscillator, they do agree). We discuss how to compare classical to quantum motion in this new paradigm, which is quite similar, but different, from the conventional approach.

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[1] Jason Tran, Leanne Doughty, and James K. Freericks, The right way to introduce complex numbers in damped harmonic oscillators, Am. J. Phys. 93, 437-440 (2025). Doi: 10.1119/5.0220797

[2] Jason Tran, Leanne Doughty, and James K. Freericks, Natural relationship between classical orbits and quantum Hamiltonians for the Kepler problem, Am. J. Phys. 93, 434-436 (2025). Doi: 10.1119/5.0220799