Topological sum rule for geometric phases of quantum gates

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We establish a topological sum rule, $\nu_U = \frac{1}{2\pi}\sum_n \gamma_n = 2m\nu_H$, connecting the geometric phases accumulated by a two-qubit system over a complete basis of initial states to the winding number ν_H classifying its Hamiltonian. Implementations of the same gate from different topological classes must distribute these phases differently, making their distinction measurable through the Wootters concurrence. As a corollary, nontrivial topology is a necessary condition for entanglement: only Hamiltonians with access to $\nu_H \neq 0$ can generate it.

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