# Structural Phase Transitions in Evolving Networks Under Different Node-Deletion Mechanisms

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Many real-world networks evolve under concurrent processes of expansion and contraction. In growing systems, preferential attachment produces scale-free degree distributions with fat tails and hubs. However, empirical evidence from declining networks shows that network contraction does not necessarily destroy structural organization: some shrinking networks maintain a stable truncated degree distribution over long periods. This raises a fundamental question: **how does the mechanism of node deletion interact with preferential attachment to determine whether hubs persist, are suppressed, or disappear entirely?** In the family of models that we consider here, at each time step a node is added with probability $P_{\text{add}}$ together with $m$ edges (according to some growth mechanism, such as preferential attachment) or deleted with probability $P_{\text{del}} = 1 - P_{\text{add}}$. The balance between growth and contraction is expressed by $\eta = P_{\text{add}} - P_{\text{del}}$, $(-1 \le \eta \le 1)$, where $\eta > 0$ corresponds to net growth and $\eta < 0$ to net contraction. In particular, we extensively studied three growth-contraction scenarios, each leading to distinct qualitative and quantitative behaviors. These can be described using closed-form analytical expressions for the degree distributions $P(k)$, which characterize the different structural phases.

**1. Random Attachment + Random Deletion (RARD):** In this scenario, random deletion does not target hubs. For $-1 < \eta < 1$, the stationary degree distribution is Poisson-like [1]. At $\eta = 1$ (pure-growth), the distribution turns exponential, indicating a structural transition.

**2. Preferential Attachment + Random Deletion (PARD):** The stationary degree distribution exhibits a power-law tail for $\eta > 0$, while for $\eta < 0$ it becomes exponential. Thus a structural phase transition occurs at $\eta = 0$ [2].

**3. Preferential Attachment + Preferential Deletion (PAPD)**: In this scenario, deletion is degree-biased and directly counteracts hub formation. The stationary distribution is power-law only in the pure-growth limit of $\eta = 1$ and becomes exponential for any $\eta < 1$ [3].

**Discussion:** These results demonstrate that the mechanism of deletion is structurally decisive. Random deletion preserves the preferential amplification of high-degree nodes, allowing scale-free structure to persist as long as growth dominates. In contrast, preferential deletion disrupts the accumulation of links by high-degree nodes, suppressing hubs even under net growth conditions. Thus, preferential deletion acts as an effective “anti-preferential attachment” force. This observation provides a theoretical explanation for the emergence of truncated or exponentially-tailed degree distributions in transient networks such as dating platforms, job-seeking platforms, and epidemic removal processes, as well as in strategically attacked networks.

[1] B. Budnick, O. Biham, and E. Katzav, *Phys. Rev. E* **106**, 044305 (2022).

[2] B. Budnick, O. Biham, and E. Katzav, *J. Stat. Mech.* 013401 (2025).

[3] B. Budnick, O. Biham, and E. Katzav, *Phys. Rev. E* **111**, 064312 (2025).