Network analysis for steady-state current fluctuations in the presence of finite affinity

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We present a graph-theoretic analysis of steady-state current fluctuations in stochastic systems described by master equations [1]. We adopt full counting statistics for both currents and dwell times to rederive the level-2.5 large-deviation function. For uniform transition rates, we perform a duality transformation and derive a signal-to-noise expression formulated as a quadratic optimization problem over mesh currents [1]. We apply this expression to the fluctuating computation time in a schematic Brownian computation process with resetting, represented by a tree-like graph with a single reset cycle. Unlike the previously derived thermodynamic uncertainty bounds, which increase logarithmically with the length of the intended computation path, this bound depends on the number of extraneous predecessors and therefore captures logical irreversibility [2].

Next, we extend the framework to arbitrary transition rates [3] by introducing a twisted incidence matrix and the corresponding modified circuit matrices, which preserve the orthogonality between the cycle space and the cut space. This allows us to perform the duality transformation in the presence of finite cycle affinity. Applying this formalism to the Brownian computation process under forward bias, we show that, in the limit of an infinitely long computation path, the Fano factor of the reset current exhibits a transition from Poissonian to nearly noiseless behavior at a forward/backward rate ratio whose threshold depends on the degree of logical irreversibility. This noise transition corresponds to the easy-hard transition in Brownian computational time complexity [4]. The transition point precisely characterizes the thermodynamic cost of logically irreversible computation: in the absence of affinity, the reset cost scales logarithmically with the length of the intended computation path, whereas reaching the transition point requires a thermodynamic force equal to the logarithm of the number of immediate predecessors per step to counteract backward branching.

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