Time Delay in Tunneling Current and the Tunneling Time

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The question *"How long does quantum tunneling take?"* has remained a longstanding problem in quantum physics. In contrast to barrier transmission (or tunneling probability), no unique theoretical or experimental definition of tunneling duration as an intrinsic property of a system has been established. Instead, it is typically inferred through the application of an external perturbation or via a specific measurement protocol.

For example, Büttiker and Landauer proposed determining the tunneling (traversal) time by applying an external oscillating field to the barrier. In the present work, we implement this idea to evaluate the average time-dependent current (conductance) through an oscillating barrier that separates two reservoirs (leads) with finite bandwidth Λ, described by the Kronig-Penney model.

We find that, in the steady state, the average conductance follows the barrier oscillations with the same frequency ω, but with a time delay ($t \to t - t_0$) arising from the traversal (tunneling) time under the barrier. A particularly simple analytical expression for this delay is obtained in the limit $\omega \to 0$. Near the band center, it reduces to $t_0 = 1/\Lambda$. In the wide-band limit (Markovian reservoirs), $t_0 \to 0$, in agreement with the classic result of MacColl, obtained by entirely different means.

Remarkably, the tunneling delay persists even in the limit $\omega \to 0$, where the barrier effectively becomes time-independent. This suggests that t_0 reflects an intrinsic property of the system, despite being revealed through an external time-periodic perturbation. We therefore propose that our result for the tunneling time delay may be applicable to a broad class of systems.

As an illustration, we consider tunneling time measurements in an optical barrier system, which provides an optical realization of the one-dimensional Kronig-Penney model. Our simple expression for t_0 shows excellent agreement with experimental data, in contrast to the Büttiker-Landauer formula for complex tunneling time applied to the same system.

Importantly, our analysis explicitly incorporates the effects of the reservoirs on the time delay. This allows us to separate these contributions from t_0 and thereby isolate the tunneling (traversal) time through the barrier. We find that the tunneling time is zero for a single barrier. Consequently, we suggest that the nonzero tunneling times observed in experiments originate from non-Markovian effects associated with finite reservoir bandwidth, and we propose ways to test this experimentally.