

Classical-Quantum Gravity as an Open Quantum System: Dilations, Decoherence and Conservation Laws

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Whether gravity must be quantized remains one of the central open questions at the interface of quantum foundations, open-system dynamics, and tests of quantum gravity. Classical-quantum hybrid theories have recently attracted attention as a possible framework in which gravity remains classical while interacting consistently with quantum matter. Schemes based on completely positive dynamics satisfy most formal consistency requirements: they preserve positivity, incorporate irreversibility and stochasticity, avoid finite-time breakdown, and enable a systematic treatment of quantum backreaction. At the same time, they invite the question of whether the hybrid description is fundamental or instead an effective sector of a larger quantum theory.

Here, we show that classical-quantum gravity based on completely positive dynamics admits a fully quantum realization on an enlarged Hilbert space. In this construction, the classical variable is represented by a commutative pointer algebra, and the hybrid evolution arises as the diagonal, operationally classical sector of a completely positive trace-preserving quantum channel, equivalently admitting a Stinespring unitary dilation. This is not a canonical quantization of the classical phase space; rather, it identifies the hybrid theory as a reduced open-system description with an imposed restriction on accessible observables.

As a complementary illustration, we consider a qubit interacting with a classical particle and demonstrate that the corresponding hybrid system violates angular momentum conservation despite rotational symmetry of the underlying equations of motion. This provides an explicit example of a fully closed, rotationally invariant classical-quantum system with completely positive dynamics that violates a conservation law. The results show that completely positive classical-quantum gravity can be embedded naturally within the framework of open quantum systems, while also demonstrating that formal consistency and symmetry alone do not guarantee the conservation laws expected of a closed physical theory.