

How to map between any two pure states with a single unitary

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It is well-known that any two pure quantum states (in the same Hilbert space) can be mapped to any other using unitary transformations. However, it was previously unknown how to determine the simplest transformation which achieves this for an arbitrary pair of pure states in arbitrary Hilbert space dimension. This contribution shows how to utilise novel algebraic methods to construct such a unitary transformation in general; and proves that the desired mapping is always possible using a single exponential of unitary Lie algebra generators. This provides a useful tool for studying the relationships between systems of pure states in quantum information theory, as well as more elementary analyses of quantum circuits.

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