Physical requirements for anomalous thermalization (Mpemba effect)

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The Mpemba effect is a thermodynamic anomaly in which a system farther away in temperature from equilibrium thermalizes before one that is initially closer. The effect has been experimentally observed across a wide range of systems, including water, colloids, and trapped ions. It has recently been the focus of numerous studies aimed at understanding its mechanisms and developing multiple applications. Despite extensive work in the field, it remains unclear which types of systems exhibit the Mpemba effect.

To address this, we derive simple necessary conditions on the transition rates for the Mpemba effect in a Markovian 3-level system. Furthermore, we analytically prove for baths at zero temperature and numerically show for the non-zero temperature case that the 3-level system conditions can be used to study the Mpemba effect in N-level systems. Multiple time scales govern thermalization in these systems. This allows the evolution to occur more quickly across larger temperature differences, explaining the Mpemba effect.

We apply our protocol to evaluate which types of systems exhibit the Mpemba effect and, in doing so, explain why the Mpemba effect in Markovian systems remains a thermodynamic anomaly. In particular, we found that the Mpemba effect requires transition rates that increase with the energy difference. Due to the maximum entropy principle, for bosonic baths the thermal component of the transition rates decays with the energy difference, limiting the bath spectra that can produce the Mpemba effect. This allows us to rule out the N-level system Mpemba effect for sub-Ohmic, Ohmic, and for some super-Ohmic spectra. These spectra describe a wide range of physical and chemical phenomena that will not exhibit the Mpemba effect. Moreover, our results provide a clear path to determine the minimal physical requirements for the Mpemba effect, and we use them to understand its underlying mechanisms better. Finally, our protocol could help identify relevant parameters for experiments, numerical simulations and diverse applications.

[1] I. Avitan, R. Factor, and D. Gelbwaser-Klimovsky. arXiv:2603.04567 (2026).