Theorem on von Neumann entropy conjectured from physics of irreversibility

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Emergence of macroscopic irreversibility from the underlying microscopic dynamics is a fundamental puzzle in theoretical physics. In a Boltzmann gas under local reversible perturbation, the restoration of indistinguishability of the molecules (i.e., a non-equilibrium generalization of Gibbs famous factor 1/N!) turns out to generate microscopic entropy in exact coincidence with the thermodynamic entropy. A similar result holds in the abstract quantum gas, if we impose symmetrization of the perturbed composite state of the constituents — in quantum informatics, it is called twirling over the group of permutations of the constituents. Furthermore, in Gibbs quantum state of a homogeneous system, the thermodynamic entropy production of a local perturbation will be exactly underlined by the von Neumann entropy produced by twirling over the group of translations. Whether twirlings are the real and ultimate way for Nature to 'forget' some particular microscopic data remains an open question.

The coincidence of microscopic and thermodynamic entropies relies on an asymptotic relationship for von Neumann relative entropy, which was conjectured first by physicists (D. Feldmann, Kosloff) assuming the said coincidence of entropies, and was proved by mathematicians (Csiszár, Hiai, Petz). Imre Csiszár told me: I cannot see how you came to this conjecture. I answered: Oh, and I cannot follow how you proved it.