The Quantum Plumber's Problem (or Quantum Advantage in Interaction-Free Interferometric Which-Path Discrimination)

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Recent work quantified the notion of quantum counterfactual gain—the advantage quantum interaction-free measurement gives over classical (un-interfering particle-type) strategies for identifying the presence or absence of an absorber in a given path. Using a recent analysis of the relation between states in the five measurement contexts for a three-level system, we here extend this work to identifying quantum advantage in a new scenario, which we term the "quantum plumber's problem". In this scenario, we imagine a "quantum plumber", who knows that one path of an interferometer is blocked, but wants to find the optimal strategy for identifying which path this is. We quantify the advantage that the use of quantum resources (beyond either interference-free particles or weak coherent states) gives the plumber, and present strategies for determining the optimal input state for the interferometer for performing such a task. We further link this to recent analysis showing to what extent such a three-path interferometer can and cannot be represented using either Quantum Simulation Logic (based on Spekkens' toy model) or the stabiliser subset of quantum mechanics, and use this to link advantage in this game to "magic" (or "non-stabiliser-ness") in the interferometer and initial state.

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