Pilot-wave hydrodynamics

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In 2005, Yves Couder discovered that millimetric droplets may self-propel along the surface of a vibrating liquid bath, guided by their Faraday pilot-wave field. Subsequent studies have shown that these walking droplets exhibit features previously thought to be exclusive to the quantum realm, including single-particle diffraction and interference, quantized orbital states, probabilistic tunneling, statistical projection effects in corrals, Anderson localization, interaction-free measurement and violations in static Bell tests [1]. The key feature of the walking-droplet system is that it is non-Markovian: the bath serves as the memory of the droplet, and all of its emergent quantum features arise in the long-memory limit where its wave field is most persistent.

The walking-droplet system represents a macroscopic realization of the pilot-wave dynamics proposed for microscopic quantum particles by Louis de Broglie in his theory of the double-solution. Both systems are characterized by two distinct wave forms, a monochromatic pilot-wave field generated by the particle vibration, and an emergent wave form describing the particle statistics. In each of the hydrodynamic analogs, the droplet's bouncing frequency plays the role of the particle's Compton frequency in de Broglie's mechanics, and the Faraday wavelength plays the role of the de Broglie wavelength in its quantum counterpart [2]. Experimental and theoretical explorations allow us to assess the potential and limitations of the walking-droplet system as a quantum analog. Particular attention is given to illustrating how the non-Markovian droplet dynamics may give rise to features that are typically taken as evidence of quantum nonlocality in their microscopic counterparts.

Theoretical descriptions of pilot-wave hydrodynamics provide a point of comparison for existing quantum pilot-wave theories. We demonstrate that the walking droplets are more closely aligned with de Broglie's original conception, his double-solution pilot wave theory, than the subsequent de Broglie-Bohm theory. Our theoretical description has also allowed us to explore a broader class of classical pilot-wave systems. For example, extending our classical model to three dimensions has revealed the spontaneous emergence of classical spin states, in which the particle translates along a helical path that is quantized in both radius and angular momentum [3]. Finally, we compare the wave form accompanying the walking droplets to the wavefunction of an exciton-polariton. Since both systems are driven and dissipative, we are able to draw a direct mathematical map between the two systems, thereby rationalizing their strikingly similar wave forms.

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