

Toward Algebraic Quantum Control

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The ability to exactly obtain the dynamics of a physical system is a core goal of most areas of modern physics. Full knowledge of the state of a system at all times would be of greatest advantage for a myriad of tasks of current interest, such as quantum simulation, computing, and control. Quantum dynamics are characterized by the non-commutativity of operators in the Hamiltonian, which in turn implies that analytical solutions are, in general, impossible to obtain. A standard way to approach this problem is to use of ad-hoc solutions or numerical techniques. While this allows for a better understanding of the physical processes of interest, such understanding remains only partial, and the question of how to obtain full control over the dynamics remains open.

We introduce a novel approach to determine the dimensionality of a Hamiltonian Lie algebra of interacting bosonic systems by appropriately classifying the space of its generating terms, thereby dividing the space of arbitrary linear Hermitian operators into classes with a meaningful physical interpretation. A first main result on the constraints that must be fulfilled by Hamiltonians without drift in order for the Hamiltonian Lie algebra to be finite-dimensional is obtained. Extension of this work to the classification of such algebras for one self-interacting bosonic mode is also provided. A generalization of the main result for arbitrary modes is obtained, thus tackling a wide range of physically-relevant scenarios. Our work has important implications for theoretical (quantum) physics as well as the theory Lie algebras.