Contradictory predictions for the Casimir force under Dirichlet-Neumann boundary conditions: Gaussian versus mean-field model

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The mean-field model (MFM) is the workhorse of statistical mechanics: one normally accepts that it delivers results that, despite being numerically just close to the correct ones, are not ”very wrong,” i.e., they resemble the actual behavior of the system as can be eventually obtained by more advanced treatments. For example, this is true for the Casimir force under Dirichlet-Dirichlet (+,+) and (+,-) boundary conditions (BC), where the MFM is expected to deliver an attractive force for like BC or a repulsive force for unlike BC, with the maximum strength of the force positioned either below or above the critical point T_c. It turns out, however, that this is *not* the case with Dirichlet-Neumann (DN) BC. For them, the mean-field approach leads to an attractive Casimir force. This contradiction with the “boundary condition rule” is cured in the case of the Gaussian model under ND BC. Our results, which are mathematically exact, demonstrate that the Casimir force within the MFM is attractive as a function of temperature T and external magnetic field h, while for the Gaussian model it is repulsive. The treatment of the MFM is based on the exact solution of one non-homogeneous nonlinear differential equation of second order. The Gaussian model is analyzed both in its continuum and as a lattice realization. The obtained outcome teaches us that the mean-field results shall cautiously be accepted when concerning the fluctuation-induced forces and ought to be checked against better treatment of the fluctuations within the envisaged system.

Partial financial support via Grant No KP-06-H72/5 of the Bulgarian National Science Fund is gratefully acknowledged.