Non-Hermitian generalization of quantum Rényi entropy

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Entropy, which is indispensable in classical and quantum channels, is one of the most important cornerstones in information theory. Non-Hermitian (NH) quantum systems attract research interest increasingly in recent years, among which the PT-symmetric, P-pseudo-Hermitian and their anti-symmetric counterpart systems are focused much more. Many meaningful results and interesting phenomena will appear when we investigate the entropy in quantum systems with NH Hamiltonians. In our work, on the one hand, we extend the application of entropy to distinguish time-evolutions of different classes and phases of typical NH systems. In a general case, we show how to distinguish all the eight phases of the above NH systems step by step. On the other hand, we investigate how to describe the Rényi entropy for NH systems more appropriately. We obtain a concisely and generalized form of α-Rényi entropy, which we extend the unified order- α from finite positive real numbers to zero and infinity. Applied it to anyonic-PT symmetric systems, we reveal the continuous change of information dynamics patterns that originates from the continuity of anyonic-PT symmetry. By exploring the mathematics and physical meaning of the negative entropy in open quantum systems, we connect negative non-Hermitian quantum Rényi entropy and negative quantum conditional entropy, paving the way to rigorously investigate negative entropy in open quantum systems.

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