## Topology, Weyl physics and quartets in multi-terminal superconducting structures

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Topology ultimately unveils the roots of the perfect quantization observed in complex systems. The 2D quantum Hall effect is the celebrated archetype. Remarkably, topology can manifest itself even in higher-dimensional spaces defined by control parameters playing the role of synthetic dimensions. However, so far, a very limited number of implementations of higher-dimensional topological systems have been proposed, a notable example being the so-called 4D quantum Hall effect. In this talk show how to engineer non-trivial topological signatures like Weyl-nodes in synthetic dimensions created by multi-terminal superconductors and how Berry spectroscopy can be used to extract information about the systems quantum geometry [1]. Furthermore, I will show that mesoscopic superconducting systems can implement higher-dimensional topology and represent a formidable platform to study a quantum system with a purely nontrivial second Chern number [2]. I discuss that these systems also admit a non-Abelian Berry phase. Hence, they also realize an enlightening paradigm of topological non-Abelian systems in higher dimensions. Furthermore, such systems can host exotic topological signatures like tensor monopoles [3]. Finally, I comment on recent experimental progress to implement synthetic dimensions in semiconductor-superconductor heterostructures [4] and the analysis revealing Cooper pair quartet [4].

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