Optimal time estimation and the clock uncertainty relation for Markovian stochastic processes

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Time estimation is a fundamental task that underpins precision measurement, global navigation systems, financial markets, and the organisation of everyday life. Many biological processes also depend on time estimation by nanoscale clocks, whose performance can be significantly impacted by random fluctuations. In this work, we formulate the problem of optimal time estimation for Markovian stochastic processes, and present its general solution in the asymptotic (long-time) limit. Specifically, we obtain a tight upper bound on the precision of any time estimate constructed from sustained observations of a classical, Markovian jump process. This bound is controlled by the mean waiting time between jumps: in simple terms, the more frequently a system transitions between its underlying states, the more precisely it can function as a clock. As a consequence, we obtain a universal bound on the signal-to-noise ratio of arbitrary currents and counting observables in the steady state. This bound is similar in spirit to the kinetic uncertainty relation but provably tighter and we explicitly construct the counting observables that saturate it. Our results establish ultimate precision limits for an important class of observables in non-equilibrium systems, and demonstrate that the mean waiting time, not the dynamical activity, is the measure of freneticity that tightly constrains fluctuations far from equilibrium.