Stability in multiscale oscillatory systems away from equilibrium

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Oscillatory dynamics pervades the universe, appearing in systems on all scales. It can be studied within the frameworks of either autonomous or non-autonomous dynamics. Autonomous dynamical systems serve as mathematical models for the time-evolution of the states of *isolated* physical systems, whereas non-autonomous dynamics describes *open systems* subjected to external driving with time-varying parameters (1). While autonomous dynamics can be studied within the long-time asymptotic framework, including asymptotic stability, we will argue that this framework can be inadequate or unsuitable when investigating open systems and studying the parameter-dependence of their stability. We will provide a new framework for non-autonomous oscillatory dynamics, within which we can define intermittent phenomena such as intermittent phase synchronisation, evaluated as the stability of phase interactions (2). We will demonstrate this framework with a coupled pair of non-autonomous phase oscillators as well as a higher-dimensional system comprised of two interacting phase-oscillator networks. Counterintuitively, non-autonomous external perturbation increases the stability of perturbed oscillatory systems.

The second part of the talk will address the question how to analyse effectively time series measured from open oscillatory systems operating on multiple timescales and away from equilibrium. We will review methods that enable explicit tracking of time-localised dynamical behaviour, as opposed to the traditional framework for dynamics analysis focused on time-independent dynamical systems and based on long-term statistics (3). We will show that time-dependent oscillatory systems with only a small number of contributions may appear noise-like when analysed according to the traditional framework using power spectral density estimation. However, methods characteristic of the time-dependent finite-time-dynamics framework, such as the wavelet transform, wavelet bispectrum, or wavelet phase coherence can identify the underlying determinism and provide crucial information about the analysed system (3,4). We will present several examples from physical and living systems, including the ageing brain.

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