# On ensemble dependence of fluctuation-induced forces: Exact results for Casimir and Helmholtz forces

Daniel M Dantchev

*Institute of Mechanics, Bulgarian Academy of Sciences, Akad. G. Bontchev St. bl. 4, 1113 Sofia, Bulgaria*

Fluctuations are ubiquitous; they unavoidably appear in any matter, either due to its quantum nature or due to the nonzero temperature of the material bodies and of the confined medium. If these fluctuations are correlated in space, the dependence of their spectrum on the relative positions and orientations of the bodies generates an effective force and torque, respectively, acting between them. When the degrees of freedom can enter and leave the region between the interacting objects, one speaks about the Casimir force. In the case of the electromagnetic Casimir force, the medium is the vacuum, and the underlying mechanism is the set of quantum zero point or temperature fluctuations of the electromagnetic field. The now widely-investigated critical Casimir force (CCF) results from the fluctuations of an order parameter and, more generally, the thermodynamics of the medium supporting that order parameter in the vicinity of a critical point. Recently, a review of the exact results available for the CCF has been published in Ref. [1]. In a recent Letter [2], and also in [3] we introduced the terms of a Helmholtz fluctuation-induced force and derived some results for it. It is a force in which an integral quantity of the order parameter value (say, the total magnetization) is fixed. We stress that in customarily considered applications of, say, the equilibrium Ising model to binary alloys or binary liquids, if one insists on full rigor, the case with the order parameter fixed must be addressed. In [2] and [3] via deriving there exact results on the example of Ising chain with fixed magnetization and under periodic and antiperiodic boundary conditions, we have shown that the Helmholtz force has a behavior very different from that of the Casimir force. It is interesting to note that the studied Helmholtz force has a behavior similar to the one appearing in some versions of the big bang theory: strong repulsion at high temperatures, transitioning to moderate attraction for intermediate values of the temperature, and then back to repulsion, albeit much weaker than during the initial period of highest temperature. We stress that the definition and existence of Helmholtz force are by no means limited to the Ising chain and can be addressed, in principle, in any model of interest. We note that the issue of the ensemble dependence of fluctuation-induced forces pertinent to the ensemble has yet to be studied. In the envisaged talk, we will review some recent and present some new both exact and numerical results for the behavior of the Casimir and Helmholtz forces.

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[1] D. Dantchev and S. Dietrich, Physics Reports 1005 (2023) 1-130.

[2] D. Dantchev and J. Rudnick, Phys. Rev. E 106, (2022) L042103.

[3] D. Dantchev, N. S. Tonchev and J. Rudnick, Ann. of Phys. 459 (2023) 169533.