Detailed Semiclassical Propagators for Simple but Nontrivial Systems

Stephen A. Fulling

Texas A&M University, Mathematics Dept., 3368 TAMU, College Station 77843-3368, USA

When applied to a time-dependent Schrodinger equation, the WKB method yields an approximation to the propagator (Green function) as a sum over classical particle trajectories (paths). This much is well known, but one seldom sees a semiclassical propagator worked out in any particular case. With student assistants I have been examining the details in some simple cases and finding them to be more interesting and difficult than one might expect. (1) For "a ball bouncing off a ceiling" (linearly decreasing potential with a reflecting barrier at the origin), for any choice of initial and final position and elapsed time, there are generically either two paths or none. Thus the solution is a sum of two terms, corresponding to paths that do or don't bounce off the ceiling. In some regions of phase space the approximation is improved by using initial momentum, not position, as the parameter. The rival propagators can be fairly compared by looking at Gaussian wave packets as initial data. (2) For a "soft wall" (a potential equal to 0 left of the origin and a positive power on the right), the classification of paths is more complicated but is topologically similar for all positive values of the exponent. There are 5 classes of paths. For example, if the particle starts and ends on the left side, there is always a path that stays outside, but sometimes there are two more paths that enter the right side and are kicked back out. Because acceleration in this model is never rightward, a particle cannot visit the right side more than once. For momentum initial data some calculation is needed to determine where the initial position is. In any event the action and amplitude for each path can be computed from the Hamilton-Jacobi and transport equations. Momentum-space and position-space results differ by terms of higher order in Planck's constant.