Non-orthogonal eigenvectors, fluctuation-dissipation relations and entropy production

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Celebrated fluctuation-dissipation theorem (FDT) inking the response function to time dependent correlations of observables measured in the reference unperturbed state is one of the central results in equilibrium statistical mechanics. In this work we discuss an extension of the standard FDT to the case when multidimensional matrix representing transition probabilities is strictly non-normal. This feature dramatically modifies the dynamics, by incorporating the effect of eigenvector non-orthogonality via the associated overlap matrix of Chalker-Mehlig type. In particular, the rate of entropy production per unit time is strongly enhanced by that matrix. We suggest, that this mechanism has an impact on the studies of collective phenomena in neural matrix models, leading, via transient behavior, to such phenomena as synchronisation and emergence of the memory. We also expect, that the described mechanism generating the entropy production is generic for wide class of phenomena, where dynamics is driven by non-normal operators. For the case of driving by a large Ginibre matrix the entropy production rate is evaluated analytically, as well as for the Rajan-Abbott model for neural networks.

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[1] Y. Fyodorov, E. Gudowska-Nowak, M.A. Nowak and W. Tarnowski, https://arxiv.org/abs/2310.09018