The Riemann zeta function and quantum mechanics

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The Riemann zeta function ζ plays a crucial role in number theory as well as physics. Indeed, the distribution of primes is intimately connected to the non-trivial zeros of this function. We briefly summarize the essential properties of the Riemann zeta function and then present a quantum mechanical system which when measured appropriately yields ζ. We emphasize that for the representation in terms of a Dirichlet series interference [1] suffices to obtain ζ. However, in order to create ζ along the critical line where the non-trivial zeros are located we need two entangled quantum systems [2]. In this way entanglement may be considered the quantum analogue of the analytical continuation of complex analysis. We also analyze the Newton flows [3, 4] of ζ as well as of the closely related function ξ. Both provide additional insight [5] into the Riemann hypothesis.

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