

# Ultrastrong coupling of a qubit with a nonlinear optical resonator

Fabio Mauceri, Alberto Mercurio, Omar Di Stefano, and Salvatore Savasta

*Università di Messina, Viale Ferdinando Stagno d'Alcontres, 31, Messina, 98166, Italy*

We study the interaction of a two-level atom with a single-mode nonlinear electromagnetic resonator, considering coupling strengths ranging from zero to the so-called deep strong coupling regime. When the qubit-resonator coupling is very strong, the standard Kerr model for the resonator becomes questionable. Moreover, recently, it has been shown that extra care is needed when constructing gauge-independent theories in the presence of approximations as the truncation of the Hilbert space of the matter system. Such a truncation can ruin gauge invariance leading to nonphysical results, especially when the light-matter interactions strength is very high. Here we face and solve these issues to provide a consistent nonlinear-resonator quantum Rabi model satisfying the gauge principle.