# Computation time and thermodynamic uncertainty relation of Brownian circuits

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Computers can be regarded as heat engines transforming free energy into waste heat and mathematical work [1]. The fundamental thermodynamic limit of computation has constantly attracted attention for more than four decades since the infancy of present-day computers [2]. A standard theoretical model of the thermodynamics of computation is the Brownian computer [1,3,4]. We discuss a specific class of Brownian computers, the so-called token-based Brownian circuits [4]. In this type of circuit, Brownian particles, i.e., tokens subject to thermal fluctuations, perform a random search in multi-token state space. The computation time is the first-passage time, the duration of time to reach a final state (output state) starting from an initial state (input state). The output state, i.e., the solution, is unique, although the computation time varies for each run. We numerically calculate the probability distribution of computation time of Brownian adders [4] and analyze the thermodynamic uncertainty relation [5] and the stochastic thermodynamics of error-free detections of outputs and resets [6]. The computation can be completed in finite time without environment entropy production, i.e., without wasting heat to the environment, at the cost of system entropy production by detecting outputs and resets. The signal-to-noise ratio of the computation time is below the mixed bound [5] and approximately the square root of the number of unidirectional transitions, i.e., the error-free detections and resets. The entropy production due to detections of outputs and resets is not prominent since the tokens can diffuse over the state space to approach the equilibrium uniform distribution. This contrasts with the logically reversible Brownian Turing machine, in which the entropy production increases logarithmically with the size of the state space.

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