Nonuniform convergence in moment expansions of integral work relations

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Several seminal results concerning out-of-equilibrium process take the form of an exponential average. Examples include the Jarzynski equality [1], its extension to processes with measurement and feedback [2], and more. Mathematically, the results take the form $\left\langle e^{-X}\right\rangle = e^{-B}$.

In this work [3], we highlight the importance of order of mathematical operations in exponential averages. We give physically motivated examples of parameter dependent processes in which naively using a limiting value of the parameter results in an apparent violation of the above equation. The first example is a model of a process with measurement and feedback. The singular limit of this example is that of error-free measurement. The second example is an ideal gas particle inside an infinitely (under the limit) fast expanding piston.

We show that this mathematical behavior of exponential averages is associated with nonuniform convergence of the moment series of X obtained by expanding $\left\langle e^{-X}\right\rangle$. We specify the shared characteristics of the examples. In both, the moments begin to deviate from their limiting value in high enough order, which is pushed higher as we approach the limit. This deviation grows strong in higher and higher moments. We also identify the dominant moments in the convergence of the series of moments.

[1] C. Jarzynski, Phys. Rev. Lett. 78 (1997) 2690.

[2] T. Sagawa and M. Ueda, Phys. Rev. Lett 104 (2010) 090602.

[3] H. Katznelson and S. Rahav, Phys. Rev. E 105 (2022) 024123.