Quantum trajectories for general time local master equations

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We extend quantum trajectory theory to encompass a large class of open quantum systems interacting with an environment at arbitrary coupling strength [1]. Specifically, we show that general time-local quantum master equations of the form

$$\frac{d}{dt}\rho(t) = -i[H,\rho(t)] + \sum_{l=1}\Gamma_l(t)\left(L_l\rho(t)L_l^\dagger - \frac{1}{2}\{L_l^\dagger L_l,\rho(t)\}\right)$$

with "rates" $\Gamma_l(t)$ that can take negative values, admit an unravelling in quantum trajectories with jumps. The sufficient condition is to weigh Monte Carlo averages E of state vectors by a probability pseudo-measure which we call the "influence martingale" $\mu(t)$. Concretely, the state $\rho(t)$ is reconstructed by the average

$$\rho(t) = \mathrm{E}(\mu(t)\psi(t)\psi^\dagger(t)).$$

The influence martingale satisfies a 1d stochastic differential equation enslaved to the ones governing the evolution of the state vectors $\psi(t)$. At weak coupling, the influence martingale method naturally reduces to the well-known quantum trajectory representation of the Lindblad–Gorini–Kossakowski–Sudarshan master equation. In genuine strong coupling cases, the influence martingale provides an algorithmically straightforward method to compute the evolution of open quantum systems. In contrast, to earlier methods there is no need to take memory effects into account [2] or expensive Hilbert space doubling [3]. The method places no real restrictions on the $\Gamma_l(t)$ and can therefore also simulate non-positive evolutions, for example generated by Redfield equations. Furthermore, we illustrate how our result provides a new avenue to numerically integrate systems with large numbers of degrees of freedom by naturally extending the existing theory.

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