Hyperbolic band theory

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Hyperbolic lattices are a new form of synthetic quantum matter in which particles effectively hop on a discrete tiling of two-dimensional hyperbolic space, a non-Euclidean space of negative curvature. Hyperbolic tilings were studied by the geometer H.S.M. Coxeter and popularized through art by M.C. Escher. Recent experiments in circuit quantum electrodynamics and electric circuit networks have demonstrated the coherent propagation of wave-like excitations on hyperbolic lattices. While the familiar band theory of solids adequately describes wave propagation through periodic media in Euclidean space, it is not clear how concepts like crystal momentum and Bloch waves can be extended to hyperbolic space. In this talk, I will discuss a generalization of Bloch band theory for hyperbolic lattices [1-3] and stress the intriguing connections it establishes between condensed matter physics, high-energy physics, number theory, and algebraic geometry.

[1] J. Maciejko and S. Rayan, Sci. Adv. 7, eabe9170 (2021)

[2] J. Maciejko and S. Rayan, Proc. Natl. Acad. Sci. U.S.A. 119, e2116869119 (2022)

[3] I. Boettcher, A. V. Gorshkov, A. J. Kollár, J. Maciejko, S. Rayan, and R. Thomale, Phys. Rev. B 105, 125118 (2022)