## Higher-dimensional topology in multi-terminal superconducting structures

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Topology ultimately unveils the roots of the perfect quantization observed in complex systems. The 2D quantum Hall effect is the celebrated archetype. Remarkably, topology can manifest itself even in higher-dimensional spaces in which control parameters play the role of extra, synthetic dimensions. However, so far, a very limited number of implementations of higher-dimensional topological systems have been proposed, a notable example being the so-called 4D quantum Hall effect. Here we show that mesoscopic superconducting systems can implement higher-dimensional topology and represent a formidable platform to study a quantum system with a purely nontrivial second Chern number [1]. We demonstrate that the integrated absorption intensity in designed microwave spectroscopy [2] is quantized and the integer is directly related to the second Chern number. Finally, we show that these systems also admit a non-Abelian Berry phase. Hence, they also realize an enlightening paradigm of topological non-Abelian systems in higher dimensions.

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- [2] R. L. Klees, G. Rastelli, J. C. Cuevas, and W. Belzig, Microwave spectroscopy reveals the quantum geometric tensor of topological Josephson matter, Phys. Rev. Lett. 124, 197002 (2020)