Quantum contextuality: the most general definition?

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Quantum contextuality is traditionally defined in three ways: in terms of (non)existence of joint distributions of random variables recorded in different contexts; in terms of counterfactual pre-existence of values of the random variables; and in terms of the (non)existence of hidden variables with noncontextual mapping into observable variables. All these definitions apply to systems of random variables with no disturbance, those in which random variables measuring the same property in different contexts have identical distributions. However, disturbance and contextuality are different forms of context-dependence, they can coexist within a system, and need to be conceptually separated and separately measured. This requires a generalization of the traditional definitions, one satisfying several desiderata, such as: noncontextuality should be preserved for any subsystem of a noncontextual system, it should be preserved under course-graining of the random variables, and adding or removing deterministic variables should not affect contextuality status of a system. In addition, the generalized definition should properly reduce to traditional definitions when applied to system with no disturbance. The definition proposed within the framework of the Contextuality-by-Default theory is, arguably, the most general definition satisfying these desiderata. Contextuality is defied as difference between the following two differences: (1) the difference between random variables when taken in isolation, and (2) the difference between the same random variables when taken within their contexts. The mathematical language that makes this definition rigorous is that of probabilistic couplings.

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