Machine-learning framework for customized optimal quantum state tomography

Violeta Nikolaeva Ivanova-Rohling[1,2,3], Guido Burkard[1], and Niklas Rohling[1]

[1] *University of Konstanz, Department of Physics, Universitätsstraße 10, Konstanz, Germany*
[2] *University of Konstanz, Zukunftskolleg, Universitätsstraße 10, Konstanz, Germany*
[3] *Department of Mathematical Foundations of Computer Sciences, Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Akad. G. Bonchev, block 8, 1113 Sofia, Bulgaria*

Finding the set of measurements which allows for quantum state tomography (QST) in the fastest way while reaching the desired precision is of high practical relevance. For a few scenarios, analytical solutions to this problem are known, e.g. for non-degenerate projective measurements, the eigenbases of the optimal set of measurement operators form a complete set of mutually unbiased bases (MUBs), which is known to exist for Hilbert spaces of prime-power dimension [1]; for measuring one qubit in a set of N qubits, the optimal choice for a QST measurement scheme is a set of mutually unbiased subspaces (MUSs), which can be constructed from a complete set of MUBs [2]. Here, we present a flexible scheme which allows to obtain numerically an optimized QST measurement set given the specifications of the system and of the measurements. It is applicable to many situations where no analytical expression of the optimal QST scheme is known. Its physical relevance is demonstrated by considering the qubit-qutrit system of dimension six where no complete set of MUBs is known. Such a system is realized for example in a nitrogen-vacancy center in diamond by the N-14 nuclear spin-1 (qutrit) and two electronic states (qubit). We formulate the search for the fastest QST measurement scheme as a high-dimensional optimization problem and numerically obtain a solution whose deviations from a set of MUSs are so small that they are without practical relevance [3]. Furthermore, we investigate the performance of machine learning approaches applied to the optimization problem we obtain for the case of individual rank-1 projection operators being the measurements in an eight-dimensional system [4]. We demonstrate the usefulness of machine learning techniques to discover measurement schemes, which are significantly better than the ones discovered by standard numerical methods. The high-performing quorums of projection operators we have discovered have complex structure and symmetries. This work was partially supported by the Zukunftskolleg (University of Konstanz) and the Bulgarian National Science Fund under the contract No KP-06-PM 32/8.

[1] W. K. Wootters and B. D. Fields, Ann. Phys. 191, 363 (1989).

[2] B. G. Bodmann and J. I. Haas, Proc. Amer. Math. Soc. 146, 2601 (2018).

[3] V. N. Ivanova-Rohling, G. Burkard, and N. Rohling, arXiv:2012.14494.

[4] V. N. Ivanova-Rohling and N. Rohling, Cyb. and Inf. Technol., 20 (6), 61 (2020).