# Discrete Time Crystals in Ultracold Quantum Gases

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A discrete time crystal (DTC) is an example of non-equilibrium quantum system which exhibits discrete time translation symmetry breaking, where measurable quantities such as the position probability density have periodicities that are an integer multiple of the period for that of the Hamiltonian [1].

We present a fully comprehensive multi-mode quantum treatment based on the truncated Wigner approximation (TWA) [2] to study many-body effects and effects of quantum fluctuations on the formation of a discrete time crystal in a Bose-Einstein condensate (BEC) bouncing resonantly on a periodically driven atom mirror [1]. Zero-range contact interactions between the bosonic atoms are assumed. Our theoretical approach avoids the restrictions both of mean-field theory, where all bosons are assumed to remain in a single mode, and of time-dependent Bogoliubov theory, which assumes boson depletion from the condensate mode is small. We show that the mean-field and time-dependent Bogoliubov approaches can be derived as approximations to the TWA treatment. Non-zero temperature BECs can also be treated.

For realistic initial conditions corresponding to a harmonic trap condensate mode function, our TWA calculations performed for period-doubling agree broadly with recent mean-field calculations [1, 3] for times out to at least 2000 mirror oscillations, except at interaction strengths very close to the threshold value for DTC formation, where the position probability density differs significantly from that in mean-field theory. For typical attractive interaction strengths above the threshold value for DTC formation and for the chosen trap and driving parameters, the TWA calculations indicate a quantum depletion due to quantum many-body fluctuations of less than about two atoms out of a total of 600 atoms at times corresponding to 2000 mirror oscillations, in agreement with time-dependent Bogoliubov theory calculations [3]. On the other hand, for interaction strengths very close to the threshold value for DTC formation, the TWA calculations predict a large quantum depletion - as high as about 260 atoms out of 600. We also show that the mean energy per particle of the DTC does not increase significantly for times out to at least 2000 mirror oscillations, typically oscillating around an average value close to its initial value; so TWA theory predicts an absence of thermalisation. Finally, we find that the dynamical behaviour of our system is largely independent of whether the boson-boson interaction is attractive or repulsive.

[1] K. Sacha, Phys. Rev. A 91, 033617 (2015); Time Crystals (Springer, Berlin, 2020).

[2] B. J. Dalton, J. Jeffers and S. M. Barnett, Phase Space Methods for Degenerate Quantum Gases (Oxford University Press, Oxford, 2015).

[3] A. Kuros et al., New J. Phys. 22, 095001 (2020).