Optimal quantum state tomography measurement set under noise

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Quantum state tomography (QST) is the process of obtaining an estimate for the density matrix of a quantum system performing measurements on several copies of the state of the system. We focus here on systems of finite dimension n. QST is an essential yet time-consuming tool for the verification of a quantum device. Consequently, finding the set of measurements which allows for QST with the largest information gain is of high practical relevance. An information-gain-based quality measure for a QST measurement set was introduced by Wootters and Fields [1]. This quality measure is the volume spanned by the linear independent projection operators included in the set of measurement. In case of non-degenerate projective measurement this was used by Wootters and Fields to show that a set of $n+1$ mutually unbiased bases is the ideal choice for the eigenbases of the measurement operators. The same quality measure can be used to obtain numerically an optimal QST measurement set for degenerate measurements, e.g. for measurements projecting on one-dimensional [2] or $n/2$-dimensional subspaces [3]. Here, we introduce a quality measure for a QST measurement set under noise, which is given by Wootters and Fields' quality measure times a noise-dependent correction factor. When the noise-affected measurement j performs the noise-free measurement with a probability q_j and project on the maximally mixed state with a probability $1-q_j$, then the correction factor is the product of the q_j to a power depending on the dimension n and on the rank of the projectors describing the ideal measurements. We use this formalism to investigate (i) QST for a single qubit with noise being zero at the poles and maximal at the equator of the Bloch sphere, (ii) QST for two qubits with non-degenerate measurements, and (iii) QST for two qubits with rank-1 projectors, where the measurements in (ii) and (iii) are realized by measuring in standard basis preceded by a sequence of perfect single-qubit rotations and noisy two-qubit gates. This work was partially supported by the Zukunftskolleg (University of Konstanz) and the Bulgarian National Science Fund under the contract No KP-06-PM 32/8.

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