A renormalization-group study of interacting Bose-Einstein condensates: Absence of the Bogoliubov mode below d=4 (T>0) and d=3 (T=0) dimensions

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We derive exact renormalization-group equations for the n-point vertices ($n = 0, 1, 2, \cdots$) of interacting single-component Bose-Einstein condensates based on the vertex expansion of the effective action. They have a desirable feature of automatically satisfying Goldstone's theorem (I), which yields the Hugenholtz-Pines relation $\Sigma(0) - \mu = \Delta(0)$ as the lowest-order identity. Using them, it is found that the anomalous self-energy $\Delta(0)$ vanishes below $d_{\rm c} = 4$ ($d_{\rm c} = 3$) dimensions at finite temperatures (zero temperature), contrary to the Bogoliubov theory predicting a finite "sound-wave" velocity $v_{\rm s} \propto [\Delta(0)]^{1/2} > 0$. In other words, Bose-Einstein condensates are free from interactions at the excitation threshold. It is also shown that the one-particle density matrix $\rho(\mathbf{r}) \equiv \langle \hat{\psi}^\dagger(\mathbf{r}_1)\hat{\psi}(\mathbf{r}_1 + \mathbf{r})\rangle$ for $d < d_{\rm c}$ dimensions at finite temperatures approaches the off-diagonal-long-range-order value N_0/V asymptotically as $r^{-d+2-\eta}$ with an exponent $\eta > 0$. The exponent η at finite temperatures is predicted to behave for $d = 4 - \epsilon$ dimensions ($0 < \epsilon \ll 1$) as $\eta = 0.181\epsilon^2$. Thus, the interacting Bose-Einstein condensates are subject to long-range fluctuations similar to those at the second-order transition point, and their excitations in the one-particle channel are distinct from the Nambu-Goldstone mode with a sound-wave dispersion in the two-particle channel.

[1] T. Kita, J. Phys. Soc. Jpn. **88**, 054003 (2019).