Quantum analogue of energy equipartition theorem

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One of the fundamental laws of classical statistical physics is the energy equipartition theorem which states that for each degree of freedom the mean kinetic energy E_k equals $E_k = k_B T/2$, where k_B is the Boltzmann constant and T is temperature of the system. Despite the fact that quantum mechanics has already been developed for more than 100 years still there is no quantum counterpart of this theorem. We attempt to fill this far-reaching gap and consider the simplest system, i.e. the Caldeira-Leggett model for a free quantum Brownian particle in contact with thermostat consisting of an infinite number of harmonic oscillators. We prove that the mean kinetic energy E_k of the Brownian particle equals the mean kinetic energy $\langle \mathcal{E}_k \rangle$ per one degree of freedom of the thermostat oscillators, i.e. $E_k = \langle \mathcal{E}_k \rangle$. The symbol $\langle ... \rangle$ denotes two-fold averaging: (i) over the Gibbs canonical state for the thermostat and (ii) over thermostat oscillators frequencies ω which contribute to E_k according to the probability distribution $\mathbb{P}_k(\omega)$. The same formula holds true for a dissipative quantum oscillator and we specify conditions for validity of this relation also for other systems. We show that this relation can be obtained from the fluctuation-dissipation theorem derived within the linear response theory.