

# Exotic criticality and symmetry-protected topological states in dimerised fermion, boson and spin chain models

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Combining numerical density-matrix renormalisation group techniques and field theory we analyse the ground-state properties of several paradigmatic dimerised quantum lattice models in one dimension. First, we explore the quantum phase transition (QPT) between Peierls and density-wave (DW) states in the half-filled, extended Hubbard model with explicit bond dimerisation. We show that the critical line of the continuous Ising transition terminates at a tricritical point, belonging to the universality class of the tricritical Ising model with central charge  $c=7/10$ . Above this point, the QPT becomes first order. The entanglement spectrum shows that dimerised Peierls insulator is a symmetry-protected topological (SPT) state. By means of bosonisation we describe the transition region in terms of a triple sine-Gordon model. The field theory predictions for the power-law (exponential) decay of the density-density (spin-spin) and bond-order-wave correlation functions are in excellent agreement with our numerical data. Secondly, we consider the dimerised extended Bose-Hubbard model and show that an SPT Haldane insulator appears between dimerised Mott and DW insulating phases, at weak Coulomb interactions, for filling factor one. Analysing the critical behaviour of the model, we prove that the phase boundaries of the Haldane phase to Mott insulator and DW states belong to the Gaussian and Ising universality classes with  $c=1$  and  $c=1/2$ , respectively, and merge in a tricritical point. Furthermore we demonstrate a direct Ising QPT between the dimerised Mott and DW phases above the tricritical point. Thirdly, we demonstrate that a nontrivial SPT Haldane phase also exists in dimerised spin-1 XXZ chain with single-ion anisotropy  $D$ , up to a critical dimerisation above which the Haldane phase disappears. In addition the ground-state phase diagram exhibits large- $D$  and antiferromagnetically ordered phases. Again, for weak dimerisation, the phases are separated by Gaussian and Ising QPTs. One of the Ising transitions terminates in a critical point in the universality class of the dilute Ising model. We comment on the relevance of our results to experiments on quasi-one-dimensional anisotropic spin-1 quantum magnets.

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