Fluctuations in dissipative quantum phase transitions at finite size

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Traditionally, quantum phase transitions in driven optical systems are characterized by a sharp increase in photon number when some control parameter is changed, as exemplified by the laser and parametric oscillator thresholds [1], and the so-called spontaneous dressed state polarization [2]. Attention is increasingly drawn, however, to the strong-coupling regime [3], where changes at the one-photon level induce nonlinear effects; thus, behavior reminiscent of phase transitions is encountered with just a few photons present – i.e., at small system size. This departure from the thermodynamic limit is accompanied by smoothed out thresholds [4,5], where, in the absence of sharp discontinuities, a new way of characterizing transitions must be sought. In this work, we characterize the phases of two strongly coupled non-linear oscillators, where both oscillators are coherently driven and damped through their interaction with an external environment. The coupled system is shown to present three phases. Two of these phases exhibit high correlation between the cavities and are differentiated by their photon statistics. The third phase is characterized by a field localized in just one of the cavities. In order to characterize these phases, we begin from a mean-field treatment where the effects of fluctuations are neglected and a corresponding thermodynamic (large photon number) limit may be defined [6]. Steady-state solutions are found, and the steady-state photon-number imbalance and total photon number are used to define the phases of the system. The mean-field approach is then complemented by a quantum mechanical treatment, where the density matrix evolves under a Lindblad-type master equation. Departures from mean-field results are discussed, and a new way to characterize its different phases, through changes in the fluctuations, is proposed. A comparison between weak and strong coupling regimes is presented, which allows us to revisit the idea of a thermodynamic limit for dissipative quantum phase transitions of photons. The crucial role of quantum fluctuations is emphasized and quantified.

[1] H. J. Carmichael, Statistical Methods in Quantum Optics 1&2, Springer-Verlag, Berlin, Heidelberg, 1999 & 2008

[2] P. Alsing and H. J. Carmichael, Quantum Opt. 3, 13 (1991)

[3] M. Mabuchi and A. C. Doherty, Science 298, 1372 (2002)

[4] P. R. Rice and H. J. Carmichael, Phys. Rev. A 50, 4318 (1994)

[5] F. Dubin, C. Russo, H. G. Barros, A. Stute, C. Becher, P. O. Schmidt, and R. Blatt, Nature Physics 6, 350 (2010)

[6] H. J. Carmichael, Phys. Rev. X 5, 031028 (2015)