Negative records of entropy production: The infimum law

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Little is known beyond the second law about the statistics of entropy-production fluctuations in the mesoscopic world. The best insights, so far, in fluctuations of entropy production are provided by fluctuation theorems. In addition to fluctuation theorems, an important question is to understand the extreme-value statistics of entropy production. What are the statistics of records of negative entropy production (also known as infima) during a given time interval?

We derive universal equalities and inequalities on the statistics of entropy-production infima [1]. We show that the mean of the finite-time infimum of the stochastic entropy production is bounded from below by minus the Boltzmann constant. We also show that the distribution of the global infimum of entropy production is exponential with mean equal to minus the Boltzmann constant. Key to our work is the connection between entropy production and martingales, which represent fair games and are widely studied in quantitative finance. The use of mathematical concepts from martingale theory for entropy production statistics bestows our results a universal character.

These results have interesting implications for stochastic processes that can be discussed in simple colloidal systems and in active molecular processes. The timing and statistics of discrete chemical transitions of molecular processes, such as the steps of molecular motors, are governed by the statistics of entropy production. We also show that the extreme-value statistics of active molecular processes are governed by entropy production; for example, we derive a relation between the maximal excursion of a molecular motor against the direction of an external force and the infimum of the corresponding entropy-production fluctuations. Using this relation, we make predictions for the distribution of the maximum backtrack depth of RNA polymerases, which follow from our universal results for entropy-production infima.

[1] I. Neri, É. Roldán, and F. Jülicher, Physical Review X 7 , 011019 (2017).