Quantum root mean square error and universally valid uncertainty relations

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Recently, the problem of extending the classical notion of root mean square (rms) error, originally introduced by Gauss [1], to quantum measurements has attracted considerable attention [2,3]. To set a theoretical basis for this problem, here, we introduce the following basic requirements to be satisfied by any useful quantum generalizations of the classical rms error: (i) device-independent definability, (ii) correspondence principle (requiring coincidence with the classical notion in the commutative case), (iii) soundness (requiring to take zero for precise measurements [4]). We show that the root mean square of the noise operator, a notion having been used for a long time, satisfies all the above requirements, whereas the recently proposed [3] error notion based on distance between probability measures does not satisfy the correspondence principle. We show a simple method to strengthen the noise-operator based rms error to satisfy (iv) completeness (requiring to take zero only for precise measurements) in addition to the above three requirements. Recently obtained universally valid measurement uncertainty relations [5,6,7,8,9,10] are maintained with the same forms by this completion of the noise-operator based rms error. This clears a recent claim [3] that the state-dependent formulation of measurement uncertainty relations is not tenable.

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