Mutual information: A key concept for irreversibility and mesoscopic thermodynamics

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Mutual information is a key concept in both classical and quantum information theory. In the context of thermodynamics, it can be used to quantify correlations between subsystems, directly characterizing the state of a composite system rather than the statistics of specific observables. In many cases, the question whether information theoretic and thermodynamic entropies are identical can be rephrased as "Can mutual information be coaxed back into observables?" Here the limitations of the spin temperature hypothesis are a good illustration of a more general question.

The "mystery of entropy production" can be reconciled with reversible microscopic dynamics using the link between global entropies, constituent entropies and mutual information: Entropy growth in a subsystem is compatible with reversible micro-dynamics if it is accompanied by an equal growth in mutual information. If this mutual information cannot be recovered by suitable probes, a process is irreversible. One might say, irreversibility is observed precisely because Maxwell's demon does not exist. Following Maxwell's philosophy of "avoiding all personal inquiries of molecules" about their past, I construct a very general effective dynamics of subsystems which "forgets" mutual information after a characteristic memory timescale. The resulting equation shares some broad features with the Boltzmann equation, which can be seen as a specific short-time limiting case of this dynamics.

The mutual information approach to irreversibility and equilibration is justified even if only *one* of the subsystems is large (and, in some broad sense, mixing): "Past" mutual information is then irrelevant and can be discarded. Equilibrium states of a *small* system interacting with a larger, reservoir-type system are thus well defined without reference to the notion of an ensemble. However, in the case of a small system, the applicable memory timescale is typically finite, i.e., mutual information in the stationary state is non-negligible. Similar results are found in in open-system dynamics beyond the Born approximation, and for the probability measure of the Gibbs ensemble of a strongly interacting composite system. Finally, taking mutual information into account leads to a natural understanding *perceived* anomalies such as negative contributions to entropy and/or heat capacity by interacting subsystems or systems strongly coupled to a reservoir.