Thermodynamic entropy as a Noether invariant

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Entropy is a fundamental concept in physics. It appears in thermodynamics, statistical mechanics, information theory, computation theory, and thermodynamics of black holes. Recently, the inter-relation between different types of entropy has been discovered. By synthesizing various aspects of entropy, we thus obtain a deeper understanding of fundamental laws in physics. Now, there is a paper [1], which claims that black hole entropy is obtained as the Noether charge associated with the horizon Killing field. We are then naturally led to ask whether thermodynamic entropy of standard materials is also characterized by a Noether invariant. In my talk, we first study a classical many-particle system with an external control represented by a time dependent parameter in a Lagrangian. We show that thermodynamic entropy of the system is uniquely characterized as the Noether invariant associated with a symmetry for an infinitesimal non-uniform time translation, where trajectories in the phase space are restricted to those consistent with quasi-static processes in thermodynamics [2]. The most remarkable result of our theory is the emergence of a universal constant of the action dimension, while our theory stands on classical mechanics, classical statistical mechanics, and thermodynamics. Next, we study a thermally isolated quantum many-body system with an external control represented by a time-dependent parameter. By formulating a path integral in terms of thermal pure states, we derive an effective action for trajectories in a thermodynamic state space, where the entropy appears with its conjugate variable. In particular, when operations are quasi-static, the symmetry for the uniform translation of the conjugate variable emerges in the path integral. This leads to the entropy as a Noether invariant [3].

[1] R. M. Wald, Phys. Rev. D 48, R3427 (1993)
[2] S. Sasa and Y. Yokokura, Phys. Rev. Lett. 116, 140601 (2016)
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