# Quantum field theory on fractals – from spontaneous emission with a fractal QED vacuum to quantum gravity at the Planck scale. Recent results and experiments.

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Fractals define a new and interesting realm for a discussion of basic phenomena in quantum field theory (QFT) in general and in QED in particular. This interest results from specific properties of fractals, e.g., their dilatation symmetry as opposed to the translation symmetry of Euclidean space and the corresponding absence of Fourier mode decomposition. Moreover, the existence of a set of distinct (usually non integer) dimensions characterizing the physical properties (spatial or spectral) of fractals make them a useful testing ground for dimensionality dependent physical problems.

After presenting general features of QFT on fractals, we shall discuss in more details basic QED physics such as spontaneous emission of a quantum emitter coupled to a fractal vacuum. We will present and discuss recent experimental results.

We shall then turn to the case of massive bosons and discuss the nature of Bose-Einstein condensation and the onset of superfluidity in fractal structures. The existence of distinct fractal dimensions characterizing spatial and spectral properties is instrumental in understanding the dimensionality dependence of the BEC and the existence of a superfluid order either through the existence of an "Off Diagonal Long Range Order" (ODLRO) or the generalization of the Mermin-Wagner theorem on long range order and its implication on the existence of topological defects.

Finally, we shall present recent results obtained in quantum Einstein gravity and discuss short scale fractal structure of the quantum universe.

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