# Experimental test of universal complementarity relations

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The principle of complementarity, considered by Niels Bohr to lie at the heart of quantum theory, asserts that the respective experimental arrangements for accurately measuring two quantum observables are, in general, physically incompatible. Thus, it restricts the degree to which *joint* information about the observables can be obtained, from a single experimental setup.

The complementarity principle is quantified by *complementarity relations*, which limit the accuracy with which two observables $A$ and $B$ can be simultaneously measured. For example, suppose that a simultaneous measurement of two compatible observables, $A_{\rm est}$ and $B_{\rm est}$, is used to estimate $A$ and $B$, and define the *inaccuracy* of $A_{\rm est}$ by the root-mean-square error $\epsilon(A_{\rm est}) := \langle (A_{\rm est} - A)^2 \rangle^{1/2}$. If the estimates are 'globally unbiased', i.e., $\langle A_{\rm est}\rangle = \langle A\rangle$ and $\langle B_{\rm est}\rangle = \langle B\rangle$ for all states, then [1]

$$\epsilon(A_{\rm est})\,\epsilon(B_{\rm est}) \geq c := |\langle [A,B]\rangle|/2. \tag{1}$$

However, Eq. (1) is not universally valid. For example, it fails for joint estimates of the position and momentum of a particle that shares Einstein-Podolsky-Rosen (EPR) correlations with a second particle [2].

More recently, universally-valid complementarity relations have been obtained by Hall [2] and Ozawa [3], such as $\epsilon(A_{\rm est})\,\epsilon(B_{\rm est}) + \epsilon(A_{\rm est})\,\Delta B_{\rm est} + \Delta A_{\rm est}\,\epsilon(B_{\rm est}) \geq c$. This relation implies one can tailor joint estimation schemes to a specific state of interest, so as to reduce the inaccuracies. However, the *spreads* of such tailored estimates must be correspondingly large. The relation is saturated by suitable position and momentum estimates on Gaussian EPR states [2].

Here the first experimental test of such universally-valid relations—as well as of a new and stronger complementarity relation—is reported, in a scenario in which the Arthurs-Kelly relation (1) is violated [4]. It exploits EPR-type correlations between two photonic qubits, to simultaneously estimate complementary polarisation observables $X$ and $Y$ of one of the qubits. To determine the inaccuracies that appear in the complementarity relations, a new method is used that does not rely on state tomography or weak measurements, but on a semiweak measurement of arbitrary strength.

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[2] M. J. W. Hall, *Phys. Rev. A* **69**, 052113 (2004).

[3] M. Ozawa, *Phys. Lett. A* **320**, 367 (2004).

[4] M. Weston et al., arXiv:1211.0370 [quant-ph].