Thermodynamic anomaly and reentrant classicality of a damped quantum system

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We study thermodynamics of open quantum systems based on the reduced partition function. We are interested in general features pertaining to linear environments with spectral densities of the coupling proportional to ω^s at low frequencies, and algebraic cut-off at high frequencies. The low-energy features of a damped system are determined by the low-frequency characteristics of the Laplace transform $\hat{\gamma}(z)$ of the damping kernel. Generally, we have $\hat{\gamma}(z) = [\gamma/\sin(\pi s/2)]z^{s-1} + (\Delta M/M)z$, where the first term describes damping, and the second term conveys mass renormalization. For super-Ohmic baths with $s > 2$, the dressed mass $M_{\rm dr} = M + \Delta M$ is generally larger than the bare mass M, whereas in the regime $0 < s < 2$ the dressed mass is always smaller than the bare mass, and even becomes negative when the damping parameter γ exceeds a critical damping strength. The fact that ΔM is negative for $s < 2$ largely escaped notice, since the term $(\Delta M/M)z$ is usually a sub-leading contribution to $\hat{\gamma}(z)$. Interestingly, mass renormalization of a free Brownian particle in the range $0 < s < 2$ is a major effect and leads to anomalous thermodynamic behavior, when $M_{\rm dr}$ falls below zero.

We present a study of the thermodynamics of a free Brownian particle. For $s < 2$, the specific heat at zero temperature increases linearly with s from $-k_{\rm B}/2$ at $s = 0$ to $k_{\rm B}/2$ at $s = 2$. Hence it is negative in the sub-Ohmic regime $s < 1$. The Ohmic bath, $s = 1$, thus represents the only case where the specific heat vanishes at zero temperature. The specific heat at low T is decreasing with T, when $M_{\rm dr}$ falls below zero, and can even become negative in the entire range $0 < s < 2$. For $M_{\rm dr} > 0$, the specific heat increases monotonically with T towards the classical value $C = k_{\rm B}/2$.

For a super-Ohmic bath with $s \geq 2$, we find a reentrant classical behavior. As the temperature is lowered, the specific heat decreases from the classical value $C = k_{\rm B}/2$, thereby indicating the appearance of quantum effects. However, the classical value $k_{\rm B}/2$ is restored, as the temperature approaches zero.

For all s, the flow into the classical regime at high temperatures is universally described by an algebraic tail of inverse power 2, $C/k_{\rm B} = \frac{1}{2} - a/T^2$, where a depends linearly on the damping parameter γ.