

Hidden assumptions in proofs of Bell

Karl Hess

Beckman Institute, University of Illinois, 405 N. Mathews Ave, Urbana, USA

John Bell's inequalities are often considered to be a cornerstone of interpretations of quantum mechanics. It is shown in this presentation that Bell's inequalities were already explained by Boole's probability theory of 1862, when he established a one to one correspondence between experimental outcomes and mathematical abstractions; two-valued functions that permit the logical operations AND OR and NOT. We now regard these functions as elements of a Boolean algebra. Violation of the inequalities indicated to Boole an inconsistency and the necessity to revise the set of mathematical abstractions that follow the rules of the algebra.

Bell and his followers derived their inequalities for two valued functions unaware of Boole's work. They attempted to link a violation of these inequalities to violations of some of Bell's assumptions regarding physical reality. It will be demonstrated that Bell's work contained also hidden assumptions, not demanded by physical law. These hidden assumptions prevented an appropriate revision of the algebra referred to by Boole. In the language of modern probability theory, Bell did not introduce general functions on the most general and appropriate sigma-algebra. These functions would have obeyed different, physically and mathematically reasonable, inequalities.

The appropriate generalization of Bell's work involves two steps. Mathematically speaking one must search for sigma-algebras, and functions on them, that remove the troublesome topological-combinatorial "cyclicities" used by Bell. Vorob'ev's work, performed 100 years after Boole and contemporary with Bell, shows how this can be done. Second, on the side of physics, and for the case of Einstein-Podolsky-Rosen (EPR) type of experiments, a possibility of space-time dependencies of preparation and/or measurement of the particles need be introduced. We will discuss such space-time dependencies based on interactions of the equipment with the environment as well as based on many body effects.

In addition to revising Bell's inequalities, our general mathematical and physical treatment permits also to assess the validity of concepts created or applied by Bell and his followers including "outcome independence" and "counterfactual reasoning". We show that Einstein local sigma-algebras and vector stochastic processes on these algebras violate outcome independence and render counterfactual reasoning irrelevant. Claims that Bell's theorem proves influences at a distance are, therefore, incorrect.