Bose-Einstein condensation, Casimir forces and quantum optics on fractal structures

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Fractals define a new and interesting realm for a discussion of basic phenomena in QED and quantum optics and their implementation. This interest results from specific properties of fractals, e.g., their dilatation symmetry as opposed to the translation symmetry of Euclidean space and the corresponding absence of Fourier mode decomposition. Moreover, the existence of a set of distinct (usually non integer) dimensions characterizing the physical properties (spatial or spectral) of fractals make them a useful testing ground for dimensionality dependent physical problems.

We shall start by noting that the absence of Fourier transform on a fractal implies necessarily different notions of volume in direct and reciprocal spaces and thus the need to modify the Heisenberg uncertainty principle. Implications for field quantization and the definition of the notion of photon on a fractal will be further addressed.

These ideas will find interesting applications in quantum optics of fractal cavities. More specifically, we shall discuss the existence of a strong Purcell effect and of a modification of photon statistics as measured, e.g., in the Hanbury-Brown and Twiss setup.

We shall then turn to the case of massive bosons and discuss the nature of Bose-Einstein condensation and the onset of superfluidity in fractal structures. The existence of distinct fractal dimensions characterizing spatial and spectral properties is instrumental in understanding the dimensionality dependence of the BEC and the existence of a superfluid order either through the existence of an “Off Diagonal Long Range Order” (ODLRO) or the generalization of the Mermin-Wagner theorem on long range order and its implication on the existence of topological defects.

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